Solitons and Vortices of Shear-Flow-Modified Dust Acoustic Wave

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Abstract

Shear-flow-driven instability and a modified nonlinear dust acoustic wave (mDAW) are investigated in a dusty plasma. In the nonlinear regime a one dimensional mDAW produces pulse-type solitons and in the two-dimensional case, the dipolar vortex solutions are obtained. This investigation is relevant to magnetospheres of planets such as Saturn and Jupiter as well as dusty interstellar clouds. Here, the theoretical model is applied to Saturn's F-rings, and shape of the nonlinear electric field structures is discussed[**1**].

Introduction

- Most of the astrophysical plasma systems such as cometary tails, the interstellar medium (ISM), and magnetospheres of planets such as Jupiter and Saturn contain macroparticles (dust), neutrals (atoms, molecules), ions, and electrons.
- These plasmas are called dusty plasmas when $r_d \ll a \ll \lambda_D$, where r_d is the dust grain radius, a is the average grain interparticle distance and λ_D is the plasma Debye radius [**2**].
- In general, dust particles become negatively charged because electrons adhere to their surfaces. The presence of dust fluid in the plasma increases the complexity and many new waves are introduced in such plasmas, for example, dust acoustic (DA), dust ion acoustic and dust drift waves.
- The usual plasma waves, particularly low-frequency modes are also modified in the presence of stationary or mobile dust fluid.
- Therefore, it is interesting to investigate the linear and nonlinear dynamics of a Dust Acoustic Wave (DAW) in the presence of shear flow.

Model

- Let us consider a dusty plasma flowing along the external magnetic field $\mathbf{B} = B_0\hat{z}$ with velocity $\mathbf{v}_0 = \mathbf{v}_{i0} = \mathbf{v}_{e0} = \mathbf{v}_{d0} = v_0(x)\hat{z}$. Dust is assumed to be negatively charged and the equilibrium densities of dust, ions, and electrons respectively are n_{d0}, n_{i0} and n_{e0}.
- Furthermore, electrons and ions are assumed to be inertialess in the slow time scale of this electrostatic perturbation, and hence, both follow the Boltzmann density distributions, viz,

$$n_e \simeq n_{e0}e^{e\varphi/T_e}, \tag{1a}$$

$$n_i \simeq n_{i0}e^{-e\varphi/T_i}. \tag{1b}$$

- We consider very slow time scale perturbations $|\partial_t| \ll |\Omega_d| = (z_d e B_0/cm_d)$ with $|\partial_z| \ll |\nabla_\perp|$ and $v_{Td}k_z \ll |\partial_t| \ll k_z v_{Ti}, k_z v_{Te}$ to study the DAW which has parallel phase velocity $\omega/k_z = c_{sd}$, where $c_{sd} = (z_d T_{eff}/m_d)^{1/2}$ is dust acoustic speed and $T_{eff} = \frac{z_d n_{d0} T_e T_i}{n_{i0}T_e + n_{e0}T_i}$ is the effective temperature of the system'.

Linear Analysis

Perturbations are assumed proportional to $e^{i(k_y y + k_z z - \omega t)}$, where k_y and k_z are the perpendicular and parallel components of wavevector $\mathbf{k}$ respectively. Linearized forms of dust momentum and continuity equations gives, respectively,

$$v_{dz} = \frac{1}{\Omega_0}\left[-\frac{z_d e}{m_d}k_z\phi - \left(\frac{c}{B_0}k_y v_0'(x)\phi\right)\right] \tag{2}$$

and

$$\Omega_d^2 n_d + n_{d0}\left(\frac{c}{B_0\Omega_d}\right)k_y^2\Omega_0^2\phi + n_{d0}k_z^2\frac{z_d e}{m_d}\left(1 + \frac{v_0'}{|\Omega_d|}\frac{k_y}{k_z}\right)\phi = 0, \tag{3}$$

where $\Omega_0 = (\omega - v_0 k_z)$.
Using $n_i \simeq n_e + z_d n_d$, we obtain

$$n_d = -\frac{n_{d0}ez_d\phi}{T_{eff}} = -n_{d0}\Phi. \tag{4}$$

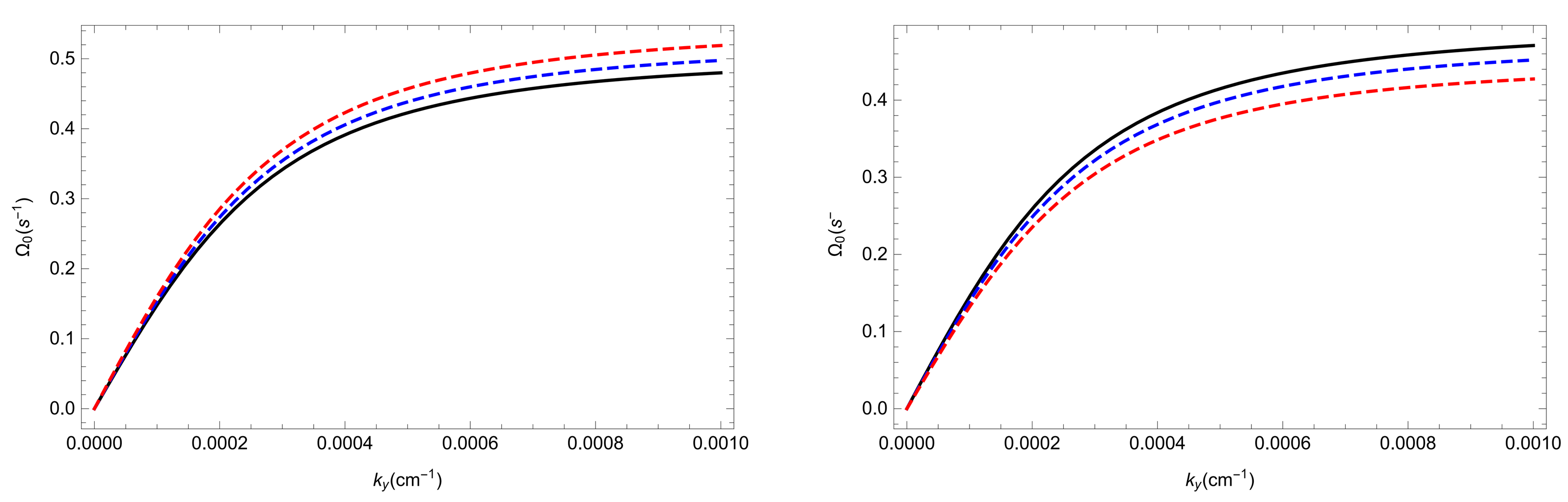
Equations (3) and (4) yield the linear dispersion relation of the DAW in a plasma with field-aligned shear flow in the form

$$\Omega_0^2 = \frac{c_{sd}^2 k_z^2}{1+\rho_{sd}^2 k_y^2}\left(1 + \frac{k_y}{k_z}S_d\right). \tag{5}$$

where $\rho_{sd} = c_{sd}/\Omega_d$ and $S_d = \frac{1}{\Omega_d}\frac{dv_0}{dx}$. Under the conditions

$$S_d < 0 \text{ and } 1 < \left|S_d\frac{k_y}{k_z}\right|, \tag{6}$$

the purely growing D'Angelo instability appears in dusty plasma. A decade ago, the shear flow-driven instability of coupled DAWs and dust drift waves was investigated. For $0 < \frac{k_y}{kz}S_d$, the frequency of the DAW is modified and instability does not arise.



(a)The Doppler shifted frequency from Eq. (12) is plotted against the wave number k_z for $S_d = 0.01$ (solid black), $S_d = 0.05$ (blue dashed curve) and $S_d = 0.1$ (red dashed curve).

(b) The Doppler shifted frequency from Eq. (12) is plotted against the wave number k_z for $S_d = -0.01$ (solid black), $S_d = -0.05$ (blue dashed curve) and $S_d = -0.1$ (red dashed curve).

Fig. 1 Plots (a) and (b) with plasma parameters of Saturn's F-ring plasma which has $B_0 = 0.1$ G, $T_e = 100$ eV, $T_i = 10$ eV, $z_d = 10^3$, $n_{d0} = 10$ cm^{-3}, and $n_{e0} = 10^3$ cm^{-3}.

Nonlinear Analysis

In this section, we analyze the nonlinear phenomena in the form of stable moving structures. The coupled differential equations are transformed into ordinary differential equations in the moving coordinates defined as $\xi = y + \alpha z - ut$, where u is the constant speed of the nonlinear structure and α is the tangent of the angle of the wave vector to the constant magnetic field $\mathbf{B}_0$.

I mDAW Soliton

Similarly as before substituting v_{dz} and n_d into the quasi-neutrality condition $\tilde{n}_d = -N_1\Phi + N_2\Phi^2$ and rearranging the terms along with the small amplitude approximation, we obtain the KdV equation,

$$-d_\xi\Phi + Ad_\xi\Phi^2 + Bd_\xi^3\Phi = 0, \tag{7}$$

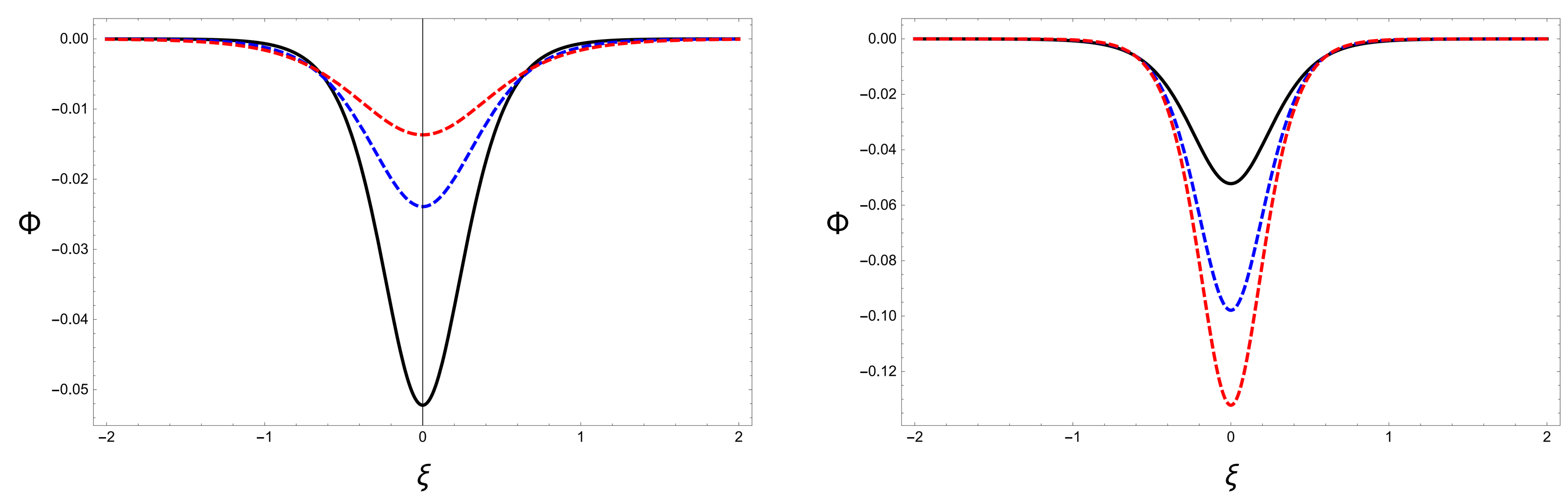
where

$$A = \frac{\rho_d}{(N_1 - \alpha b n_{d0}/a)}\left[N_2 - \frac{\alpha^2 b^2 n_{d0}}{2a^2} - \frac{\alpha N_1 b}{a}\right],\ B = \frac{\rho^2}{(N_1 - \alpha b n_{d0}/a)}. \tag{8}$$

Also where $b = \frac{c_{sd}^2(\alpha + S_d)}{a}$ and $a = (u - \alpha v_0)$, $N_1 = (\sigma n_{i0} + \tau n_{e0})/z_d$ and $N_2 = \frac{1}{2z_d}(n_{i0}\sigma^2 - n_{e0}\tau^2)$ with $\sigma = T_{eff}/z_d T_i$ and $\tau = T_{eff}/z_d T_e$. Eq. (7) describes a single pulse soliton and it admits the following solution;

$$\Phi = \Phi_0\,\mathrm{sech}^2(\xi/W). \tag{9}$$

where $\Phi_0 = 3/A$ is the maximum amplitude of the ion acoustic solitary pulse and $W = \sqrt{4B}$ is the width of the soliton.



(a) The soliton profile for different values of $S_d = 0.0$ (solid black curve), $S_d = 0.04$ (blue dashed curve), and $S_d = 0.06$ (red dashed curve) with fixed parameters $\alpha = 0.2$, $M = 0.18$ and $v_0 = 0.52$.

(b) The soliton profile for different values of $S_d = 0.0$ (solid black curve), $S_d = -0.04$ (blue dashed curve), and $S_d = -0.06$ (red dashed curve) with fixed parameters $\alpha = 0.2$, $M = 0.18$ and $v_0 = 0.52$.

Fig. 2 Plots (a) and (b) with plasma parameters of Saturn's F-ring plasma which has $B_0 = 0.1$ G, $T_e = 100$ eV, $T_i = 10$ eV, $z_d = 10^3$, $n_{d0} = 10$ cm^{-3}, and $n_{e0} = 10^3$ cm^{-3}.

II mDAW vortices

If Φ is also a function of the x-coordinate then the nonlinear wave dynamics may produce vortices. In this section, we obtain the vortex equation for an mDAW. Let us define a moving frame $\xi = y + \alpha z - ut$ and assume $\Phi = \Phi(\xi, x)$.
Continuity equation yields,

$$\left[\nabla_\perp^2 - \lambda_1^2\right]\Phi = A_0\left(\Phi - \frac{a|\Omega_d|}{c_d^2}x\right), \tag{10}$$

which can be solved using cylindrical polar coordinates to obtain vortex structures.
Let Φ be a function of (r, θ), where $r = \sqrt{x^2 + \xi^2}$ with $x = r\cos\theta$, $\xi = r\sin\theta$ and $\theta = \tan^{-1}(\xi/x)$. We assume that in the polar plane (r, θ), the electrostatic potential Φ produced by plasma nonlinearities is confined within a radius $R_0 =$ constant. We use the variable separation method and assume

$$\Phi(r, \theta) = \psi(r)f(\theta) = \psi(r)\cos\theta, \tag{11}$$

The solution of Eq. (10) for $r < R_0$ is

$$[\Phi(r,\theta)]_{\mathrm{In}} = \left[A_1 J_1(s_0 r) + \left(\frac{s_0^2 + \lambda_1^2}{s_0^2}\frac{a|\Omega_d|}{c_{sd}^2}\right)r\right]\cos\theta. \tag{12}$$

and the outer solution for $r > R_0$ is

$$[\Phi(r,\theta)]_{\mathrm{Out}} = B_1 K_1(\lambda_1 r)\cos\theta. \tag{13}$$

where $s_0^2 = -(\lambda_1^2 + A_0)$ and the constants A_1 and B_1 can be determined using the continuity of Φ, $\nabla\Phi$, and $\nabla^2\Phi$ at $r = R_0$, which yields

$$A_1 = -\frac{a|\Omega_d|}{c_{sd}^2}\left(\frac{\lambda_1^2}{s_0^2}\frac{R_0}{J_1(s_0 R_0)}\right),\ B_1 = \frac{a|\Omega_d|}{c_{sd}^2}\frac{R_0}{K_1(\lambda_1 R_0)}. \tag{14}$$

References

[1] Usman Saeed, Hamid Saleem and, Shaukat Ali Shan, J. Phys. Soc. Jpn. **87**, 014501 (2018).

[2] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics*, Taylor & Francis, Boca Raton, 2002.

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